

Galactic Models and White Dwarf Populations

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ABSTRACT

We make use of a previous well tested Galactic model, but describing the observational behavior of the various stellar components in terms of suitable assumptions on their evolutionary status. In this way we are able to predict the expected distribution of Galactic White Dwarfs (WDs), with results which appear in *rather good* agreement with recent estimates of the local WD luminosity function. The predicted occurrence of WDs in deep observations of selected galactic fields is presented, discussing the role played by WDs in star counts. The effects on the theoretical predictions of different White Dwarfs evolutionary models, ages, initial mass functions and relations between progenitor mass and WD mass are also discussed.

Key words: Galaxy:structure, Galaxy:stellar content, Galaxy:fundamental parameters, stars:white dwarfs.

1 INTRODUCTION

Since the very beginning of modern astronomy the distribution of stars over the night sky has been regarded as an evidence of the distribution in space of stellar objects, i.e., in the current terminology, as the result of the distribution of the Galactic stellar components. However, only in relatively recent times such an evidence has been used as an input for detailed investigations aiming to reconstruct the spatial distribution of the various stellar populations forming our Galaxy. Here one may recall the “classic” star counts models by Gilmore & Reid (1983, G&R, see also e.g. Reid & Majewski, 1993 and Basilio et al. 1996) and by Bahcall & Soneira (1984, B&S, see also e.g. Gould, Bahcall & Maoz, 1993) as based on the assumption of suitable spatial density distributions for the various Galactic components and on the observational luminosity function and colour-magnitude diagram for each stellar population.

In a recent paper (Castellani et al. 2001; Paper I) we revisited the argument, showing that the predictions of similar Galactic models can reach a reasonable agreement even with very deep investigations, as recently obtained by the *Hubble Space Telescope* (HST). As in the G&R and B&S works, that paper followed a semi-empirical approach, adopting, as model inputs, observational constraints on the distribution of magnitudes and colours of the various stellar populations. However, investigations of simple stellar populations in stellar clusters have already repeatedly shown the good agreement between observed and predicted CM diagrams (see, e.g., Brocato et al. 2000). This discloses that the distribution of stars, for the evolutionary phases later than MS, closely

follows current predictions as based on theoretical evolutionary lifetimes, independently of any assumption about the star Initial Mass Function (IMF). In the meantime, every adopted luminosity distribution of MS stars can be easily interpreted in terms of a corresponding suitable behavior of the IMF. Thus, each empirical assumption about the CM distribution of stellar populations can be easily transferred into suitable assumptions about their original chemical composition, IMF and age, without consequences on the Galactic model.

The use of an evolutionary input presents several advantages. One can investigate the effects of varying the adopted evolutionary parameters, constraining ages, chemical composition or IMF of the Galactic populations. Relevant steps in such a direction have been recently presented by several authors: Ng (1994), Ng et al. (1997) and Fan (1999) based their simulations on population synthesis models assuming a suitable star formation history, whereas Haywood, Robin & Créze (1997) (see also Robin & Créze 1986 and Haywood 1994) based their models on the synthesis of both the evolution of stellar population and the dynamical evolution of the vertical structure of the disc.

However, and perhaps more interestingly, for each given IMF, chemical composition and age, a theoretical isochrone gives rather firm constraints on the relative abundance of stars in all the evolutionary phases, from the MS till the final structures, either as post-supernova objects or as cooling white dwarfs. In this paper we will rely on a similar approach, transferring our Galactic models into the quoted

theoretical scenario, to derive theoretical predictions concerning the Galactic distribution of white dwarf populations.

2 THE GALACTIC MODEL

As in Paper I, we will distribute the stars according to the spatial density distribution of B&S and G&R Galactic models, but now relying on suitable assumptions on the evolutionary status and on the initial mass function (IMF) of the various Galactic populations to reproduce the luminosity functions used as an (observational) input in Paper I. Our model includes three components: spheroid, disc and thick disc. The need for an extended “thick disc” population, as formed by stars with spatial and kinematic properties intermediate between disc and spheroid, was firstly suggested by Gilmore & Reid (1983) and further supported by detailed analysis taking into account the velocity distribution of local stars (see e.g. Ojha et al. 1996, Wyse & Gilmore 1995, Norris & Ryan 1991, Casertano, Ratnatunga & Bahcall 1990). However the thick disc structural parameters (density law, local density etc.) are still debated (see e.g. Reid & Majewski 1993, Yamagata & Yoshii 1992, Ojha et al. 1996, Ruphy et al. 1996, Mendez & Guzman 1998, Buser et al. 1999, Reylé & Robin 2001).

Predicted results are obtained by randomly generating star masses according to the adopted IMF (see Sec. 2.3) and by using stellar models to derive luminosities in the selected bands for each given value of the stellar mass and age. Spheroid stars are assumed to be almost coeval and thus they are reproduced by populating a suitable isochrone, while for both thick disc and disc stars, one has to take into account prolonged episodes of star formation. Thus, for these two last components, star masses and ages are both randomly generated, the mass distribution reproducing the selected IMF, while a flat age distribution is adopted within the range assumed for each population.

In the following subsection we will shortly describe the main theoretical ingredients of our model producing the distribution of the observed nuclear burning luminous structures. The corresponding predictions for WDs will be presented and discussed in sections 3 and 4.

2.1 Evolutionary tracks

The code relies on a set of homogeneous evolutionary computations covering both the H and He burning phase for stars with original masses in the range $0.1 \div 7 M_{\odot}$. Very low MS tracks are from evolutionary calculations by Cassisi et al. (2000), which are already shown to be in good agreement with recent theoretical calculations, as well as with HST observations of faint MS in Galactic Globulars (see e.g. Cassisi et al. 2000) and with recent data by Monet et al. (1992) and Dahn et al. (1995) for subdwarfs stars in the solar neighborhood (see e.g. Paper I). Colour transformations are from Allard et al. (1997) model atmospheres. However, as discussed in Baraffe et al. (1995) and in Brocato et al. (1998), theoretical models do not satisfactory reproduce the observed distribution of very low mass (VLM) stars at solar metallicity. Even if such a disagreement does not affect the WD predictions we will deal with, to present a model as reliable as possible, for stars with $M \leq 0.6 M_{\odot}$ and solar metallicity

we used the empirical V-(V-I) data by Monet et al. (1992) and Dahn et al. (1995) and the observational LF by Wielen, Jahreiss & Kruger (1983). For $M > 0.6 M_{\odot}$ we use the Cassisi et al. (1998) and the Castellani, Degl’Innocenti & Marconi (1999) evolutionary tracks up to the end of the AGB, which are in excellent agreement with recent results from the Hipparcos satellite for nearby stars (Kovalevsky 1998) and open clusters (see e.g. Castellani, Degl’Innocenti & Prada Moroni 2001, Petroni 1999) and with observational data of globular clusters at different metallicities (see e.g. Cassisi et al. 1999, Brocato et al. 2000). The adopted colour transformations are from Castelli et al. (1997).

2.2 IMF

Several authors (see e.g. Scalo 1998, Kroupa 2000) discussed in details local star counts in order to infer the underlying IMF. While the most of authors agree on a Salpeter IMF for $M \gtrsim 0.5 M_{\odot}$ (see e.g. Massey et al. 1995, Kroupa 2001a) the IMF behaviour for lower masses is still uncertain (see e.g. Scalo 1998, Kroupa 2001a, Reylé & Robin 2001). The uncertainty on the IMF for low mass stars reflects the observational errors on the local luminosity function (LF) at low luminosity for disc and spheroid stars (see e.g. Gould, Flynn & Bahcall 1998, Paper I, for a discussion). Moreover, some authors (Scalo 1998, Kroupa 2001b, Eisenhauer 2001) suggest that the IMF could vary with time and with environment, an hypothesis which is rejected by other researchers (see e.g. Gilmore 2001).

In our model we assume the “average Galactic field IMF” proposed by Kroupa (2001b) for the three Galactic components, that is $dN/dM \propto M^{-\alpha}$ with :

$$\begin{aligned} \alpha &= 1.3(\pm 0.5) & \text{for} & \quad 0.08 \leq m/M_{\odot} < 0.5 \\ \alpha &= 2.3(\pm 0.3) & \text{for} & \quad 0.50 \leq m/M_{\odot} \end{aligned}$$

As a whole, with the adopted IMF and evolutionary tracks, theoretical luminosity functions appear in agreement with the observed ones within their observational uncertainties.

2.3 Spheroid

As usually, we adopt for the spheroid the de Vaucouleurs density law. The assumed Galactocentric distance is $R_0 = 8$ Kpc and the axis ratio for the spheroid is 0.8 (see Robin, Reylé & Créze 2000, for a discussion). Following Gilmore & Reid (1983, see also e.g. Basilio et al. 1996) we adopt the halo/disc density ratio of 0.125%. A theoretical isochrone of 12 Gyr ($Z=0.0002$, $Y=0.23$), populated by using the Kroupa (2001a) IMF, is assumed. The chosen theoretical approach is spontaneously producing predictions about the occurrence of stars in the various evolutionary phases. As an example, Fig. 1 shows the predicted CM diagram on a selected area of 1 square degree at the North Galactic Pole (NGP) with the contribution by subgiant (SGB) and red giant (RGB) stars. The only one HB star predicted in that field is not plotted, since no firm theoretical constraints exist for its colour.

2.4 Disc

The adopted disc density law is the widely used double exponential in the Galactocentric distance on the Galaxy plane and in height over the plane, with scale length and scale

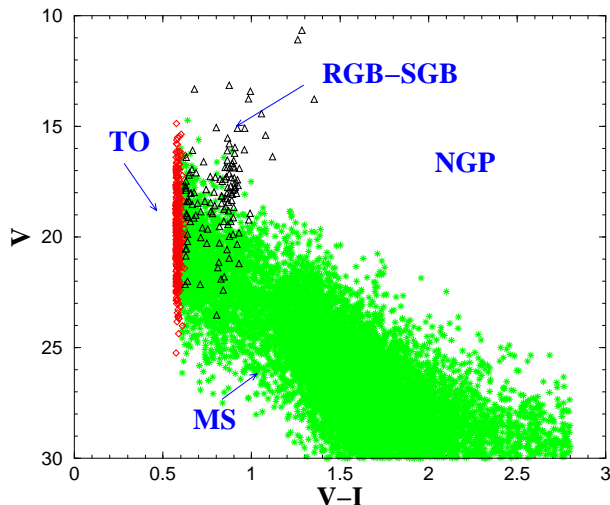


Figure 1. Theoretical CMD for spheroid stars in a field of area 1 square degree at the North Galactic Pole (NGP). Different symbols represent stars in the various evolutionary phases.

height of 3500 pc and 325 pc, respectively (Reid & Majewski 1993, Basilio et al. 1996). We assume a constant star formation rate (SFR) from 50 Myr to 9 Gyr to populate evolutionary tracks with $Z=0.02$ $Y=0.27$.

2.5 Thick disc

The thick disc density law can be reasonably modeled either by a double exponential or by a density law close to $\text{sech}^2(z)$. Star counts are unable to distinguish between these two hypotheses (Reyl   & Robin 2001) and the thick disc structure is generally described as a double exponential, with horizontal and vertical scales approximately in the ranges $r_0 \approx 2.5 \div 4.0$ Kpc, $z_0 \approx 600 \div 1600$ pc, respectively. The scale length and height and the thick disc/disc density ratio suggested by Gilmore & Reid (1983) (see also e.g. Basilio et al. 1996) and adopted in this paper are 3500 pc, 1300 pc and 2%, respectively. Following Gilmore, Wyse & Jones (1995) and Norris (1999) we choose for the thick disc a metallicity of $\sim Z=0.006$ and a SFR centered at ~ 10 Gyr with a spread of few Gyr.

2.6 Reddening/obscuration

We adopt for intermediate-to-high Galactic latitudes ($|b| \geq 10^\circ$) the E(B-V) reddening maps by Burnstein & Heiles (1982). The visual extinction $A_V = 3.12 E(B-V)$ and the reddening in the infrared colours are from Bessell & Brett (1988, see also Clementini et al. 1995). A reddening scale height of 100 pc is adopted (see e.g. Mendez & van Altena, 1998).

3 THE WHITE DWARF POPULATION

According to the discussion given in Paper I, the Galactic model presented in the previous section appears able to reasonably account for available star counts, even down to very faint magnitudes ($V \approx 27$). By relying on this model one

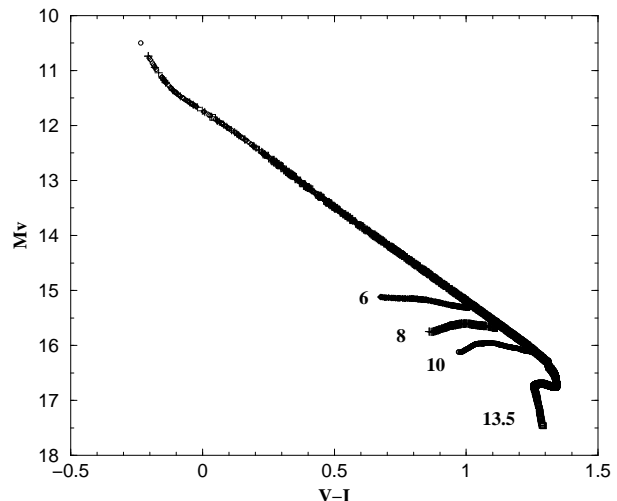


Figure 2. Theoretical isochrones in the $M_V - (V - I)$ diagram for spheroid WDs of the labeled ages (in Gyr).

can easily predict the expected abundance of WDs, since each star evolved beyond the AGB phase and less massive than the lower mass limit for supernovae (M_{up}) is assumed to be a WD. However, to predict the CM location of white dwarfs one needs further theoretical ingredients, as given by: i) a WD mass - progenitor mass relation ii) theoretical WD models giving luminosity and temperature of a WD as a function of mass and age and iii) suitable colour transformations. The WD cooling age (that is the time spent on the cooling curve) is simply given by the difference between the age associated to the star and the age of the WD progenitor at the end of the AGB. For $T_{\text{eff}} < 4000^\circ\text{K}$ we adopt the colour relations by Saumon & Jacobson (1999), which include a detailed treatment of collision induced absorption of H_2 , whereas for higher temperature, the results of Bergeron, Wesemael & Beauchamp (1995) were used. As an example Fig. 2 shows theoretical WDs isochrones.

We notice that in our model we include only WDs with hydrogen atmosphere (DA WDs) which are shown to be the most numerous. However, the last two bins of the observed disc WD LF by Legget, Ruiz & Bergeron (1998, see Fig. 7) are populated by a relevant number of non-DA WDs. As well known, it is still not clear if this phenomenon is due to a chemical evolution of the WD outer layers or to the relevant differences in the cooling times between DA and non-DA. Thus, given the present uncertainty in the theoretical scenario we prefer, as first step, do not include non-DA WDs.

The theoretical prediction of WD populations is affected by several uncertainties. First of all, theoretical models are not settled yet, as shown by the somewhat large differences among recent models in the literature (Wood 1995, Benvenuto & Althaus 1999, Hansen 1999, Salaris et al. 2000, Chabrier et al. 2000, Castellani, Prada Moroni & Straniero 2001). To investigate the dependence of the WD distribution on the adopted cooling sequences, Fig. 3 shows predicted results with two different sets of theoretical WD models, as given by Chabrier et al. (2000) and Salaris et al. (2000), which are the only models cooling down until an age of the order of the age of the Universe for a suitable range of masses. Figure 3 compares the magnitude distribu-

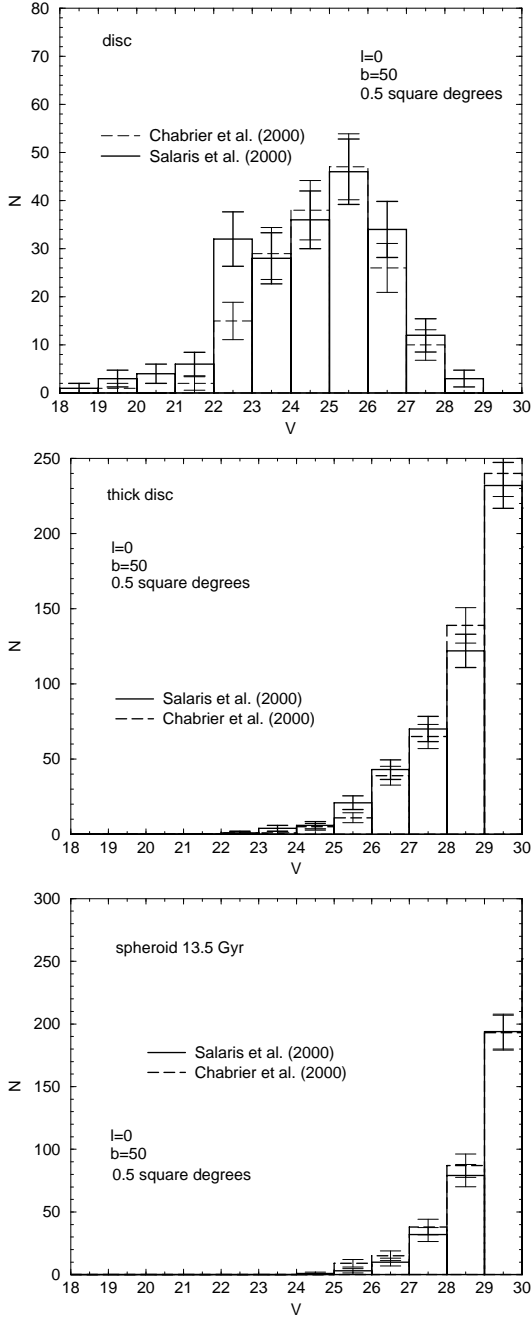


Figure 3. Comparison between the predicted V-magnitude distribution (in a field at the Galactic longitude $l = 0^\circ$ and at the Galactic latitude $b = 50^\circ$, of extension 0.5 square degrees) obtained by adopting theoretical WD cooling tracks by Chabrier et al. (2000) (solid line) and by Salaris et al. (2000) (dashed line) for the disc (upper panel), the thick disc (middle panel) and the halo (lower panel). The error bars indicate the poissonian statistical uncertainty on the counts. The WD mass - progenitor mass relation is by Weidemann (2000) for the disc/thick disc and by Dominguez et al. (1999) for the spheroid.

tion for spheroid, disc and thick disc WDs for the two sets of WD tracks; luckily enough, one finds that the differences are within the statistical uncertainties.

Another cause of uncertainty is the adopted relation between the WD mass and the progenitor mass. As well known, this indetermination is connected to the uncertainties in the final evolution of AGB stars, particularly during the thermal pulses and at the onset of the super-winds, and to the still present debate about the predicted extension of the convective core during central burning phases (see e.g. Dominguez et al. 1999). For the spheroid we adopt the theoretical relation by Dominguez et al. (1999), where the final WD mass is given by the helium core mass at the first thermal pulse, as obtained by assuming a standard extension of the convective core. For disc and thick disc one has the alternative choice between the semi-empirical relation by Weidemann (2000), as inferred from the comparison between observations of young open cluster and theoretical models and the exponential one by Wood (1992), based on the Planetary Nebula Nuclei mass distribution. Comparison between the predicted WD magnitude distribution, as given in Fig. 4, under the two alternative hypothesis, again discloses relatively small differences. Thus, we made the choice of using the theoretical relation by Weidemann (2000).

3.1 The population parameters

Before presenting more detailed predictions, and to give light on the possible variations of the adopted theoretical scenario, let us give a short discussion on the influence of the input parameters concerning the stellar population. As a first point the predicted halo WD population obviously depends on the assumption on the halo age. Figure 5 shows the predicted luminosity function at four different ages: 6, 8, 10 and 13.5 Gyrs. As well known (see e.g. Brocato et al. 1999), for a given age, the bulk of the WD distribution is close but not precisely at the faint end of the cooling isochrone. The larger is the age the fainter is the bulk of the WD population. This is a well understood feature: the increase of the time spent in the cooling sequence implies a progressive decrease of WD luminosity. Figure 5 shows the related theoretical predictions.

Furthermore, the distribution of WD populations significantly depends on the adopted IMF. The number of WDs is obviously affected only by IMF variations for masses which could evolve into WDs in a time shorter than the estimated age of the Universe. Figure 6 shows the effect on the distribution of a variation of the IMF for masses greater than $0.6 M_\odot$. As range of variation we assume the uncertainty on the Salpeter exponential, as evaluated by Kroupa (2001a); see Sec. 2.2. As expected, a steeper IMF ($\alpha=2.6$) depopulates the WD stars. This behaviour can be easily understood as a consequence of the decrease of the number of stars in the mass range able to produce WDs.

4 RESULTS

As a first test of the model, Fig. 7 compares the predicted local WD luminosity function with recent observations by Liebert, Dahn & Monet (1988) and Leggett et al. (1998); *the agreement appears good, even if it is not perfect.* We

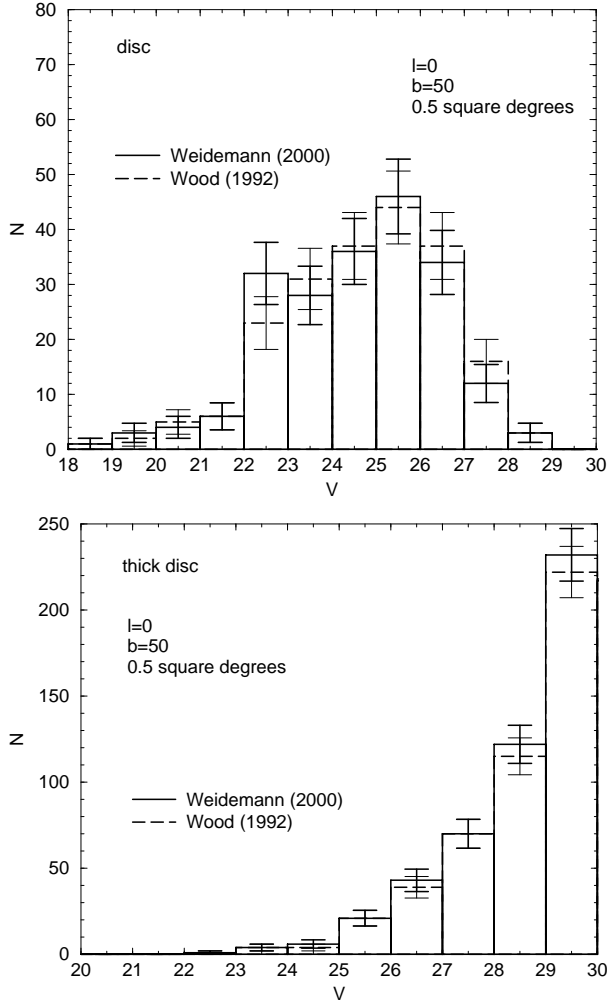


Figure 4. Comparison between the predicted V-magnitude distribution (in a field at $l = 0^\circ$, $b = 50^\circ$ of extension 0.5 square degrees) as obtained by adopting the initial-final mass relation by Wood (1992) (solid line) and by Weidemann (2000) (dashed line) for the disc (upper panel) and the thick disc (lower panel). The adopted WD tracks are the Salaris et al. (2000) ones. The error bars indicate the poissonian statistical uncertainty on the counts.

remind that the only normalization adopted in our model is the one of hydrogen-burning stars in the solar neighborhood. Thus, the density distribution of WDs naturally arises from the model and the plotted WD luminosity function is just the output of our code without any additional normalization. To reproduce the local white dwarfs luminosity function, we restricted our calculations to distances from the Sun up to 200 pc, as obtainable from Table 2 of Legget et al. (1998). The histogram of Fig.7 does not include thick disc white dwarfs because we checked that, due to the very small distance from the Sun, their contribution is negligible. Comparisons between theoretical and observed WD LFs has been presented by Legget et al. themselves by adopting WD evolutionary sequences by Wood (1995). Their fit appears similar to our results. We remind that our LF directly arises from a Galactic model built with a given spatial distribution and defined parameters, so that we do not apply any normalization to the WD LF itself. On the contrary, the Legget et

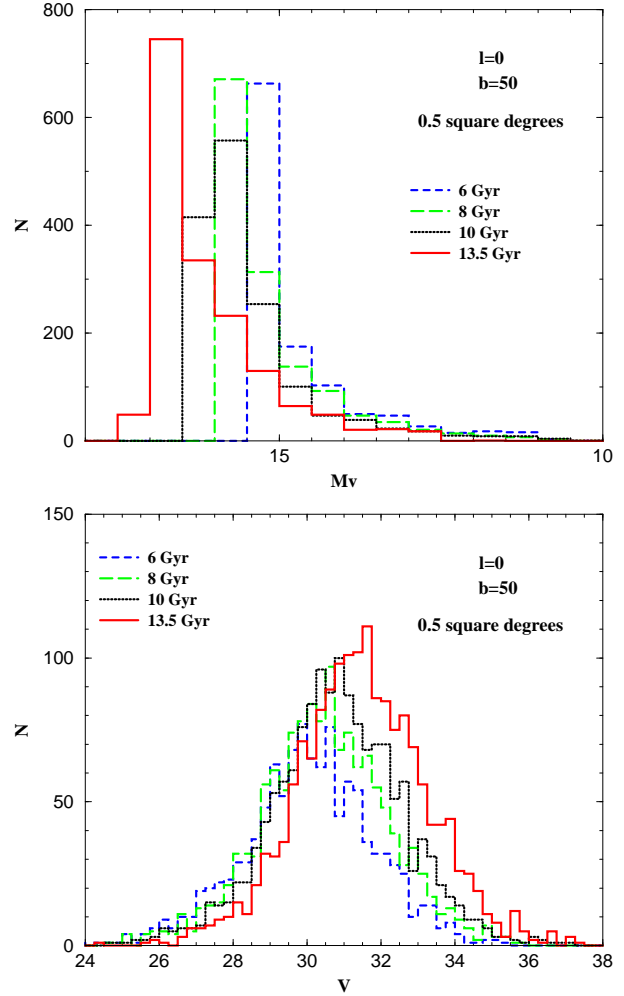


Figure 5. The predicted luminosity function (upper panel) and V-magnitude distribution (lower panel) for spheroidal WDs at different ages (in a field at $l = 0^\circ$, $b = 50^\circ$ of extension 0.5 square degrees). The adopted WD cooling tracks are from Salaris et al. (2000). The Dominguez et al. (1999) theoretical relation between the progenitor mass and the WD mass is used.

al. LF is obtained only from WD isochrones (without any assumption on the WD spatial distribution) and then it is normalized to the observational data. A detailed analysis of disc WD LFs has been presented by Hansen (1999) by using his WD isochrones. The author reproduced very well the LF profile showing the well known influence of the disc age on the cutoff region, even in this case no assumptions have been made on the Galactic spatial distributions. Also Hansen found that the inclusion of a given percentage of non-DA WDs can smooth the LF profile. It is thus possible that we could reach an even better agreement with observations by slightly changing the assumed disc age or by including non-DA WDs. Moreover even a variation of the model parameters can slightly modify the results, however a deep analysis of this problem is out of the purposes of the present paper.

By relying on the obtained agreement between theoretical and observational WD LFs, in this section will present the predictions by our model for three fields in different directions of the Galaxy.

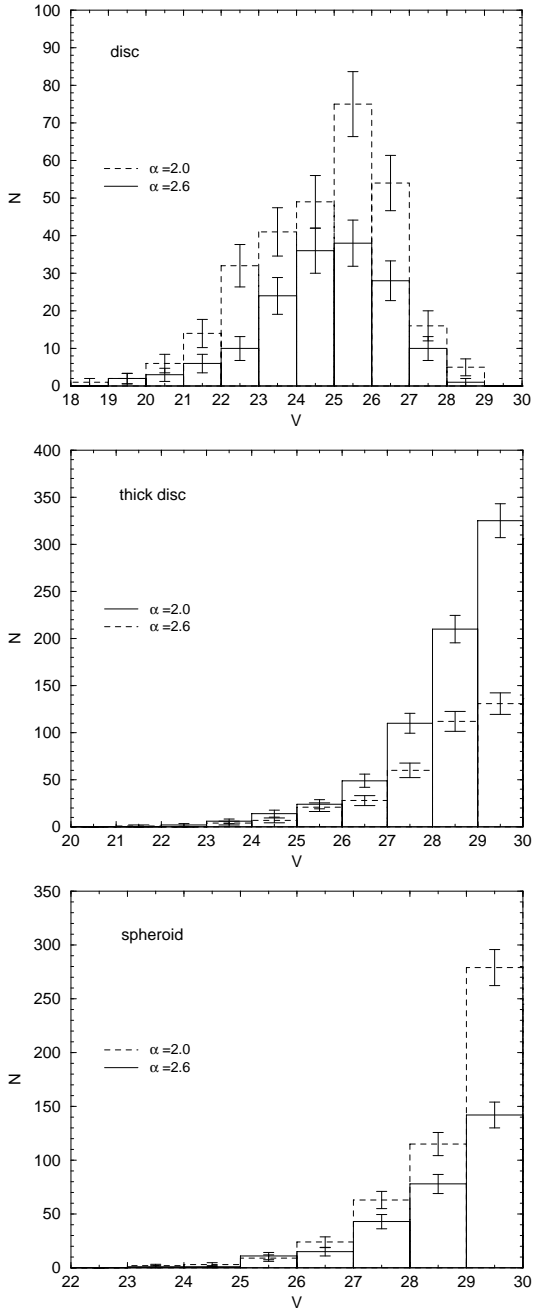


Figure 6. Comparison between the predicted V-magnitude distribution (in a field at $l = 0^\circ$, $b = 50^\circ$ of extension 0.5 square degrees) obtained by adopting, for $M > 0.6M_\odot$, as IMF exponent either $\alpha = 2.0$ (solid line) or $\alpha = 2.6$ (dashed line) for the disc (upper panel), the thick disc (middle panel) and the spheroid (lower panel). The error bars indicate the poissonian statistical uncertainty on the counts. The adopted theoretical WD tracks are from Salaris et al. (2000). The assumed age for the spheroid is 13.5 Gyr.

Future comparisons with observations will be fundamental to infer firmer constraints on the parameters assumed in the model whose values is not definitively established yet.

The $(V - I, V)$ colour-magnitude diagrams (CMDs) for the field stars at the North Galactic Pole, at the Galactic

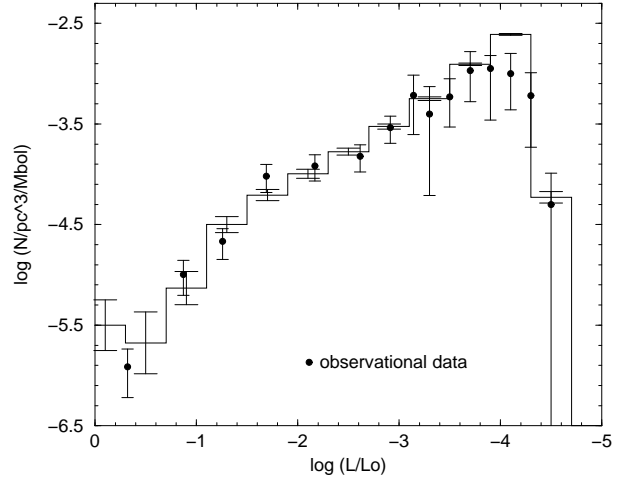


Figure 7. Comparison between the predicted local WD luminosity function and recent observations by Liebert, Dahn & Monet (1988) and Leggett, Ruiz & Bergeron (1998). Statistical (poissonian) errors in the theoretical histogram are also shown (thin lines).

coordinates $l = 0^\circ$, $b = 50^\circ$ and $l = 0^\circ$, $b = 30^\circ$ are shown in Fig. 8; the extension of each field is 0.5 square degrees. The CMDs are extended down to magnitude $V = 38$ to display the strong presence of spheroid WDs at extremely low luminosities. As discussed above, our model is able to predict various colour indices for star counts. As an example, Fig. 9 shows the $(V - R, V)$ CMD for the same field of the middle panel of Fig. 8.

The small panels in Fig. 8 show the CMDs obtained for a field of about 6.6 arcmin^2 of extension, that is the area generally covered by *Hubble Space Telescope* (*HST*) observations (see e.g. King et al. 1998). As expected, at the NGP only a negligible number of stars is present in such a small field. Star counts increases moving from higher to lower latitudes, nevertheless disc population is still absent at intermediate latitude ($b = 50^\circ$), beginning to appear only at lower latitude ($b = 30^\circ$), where there's not yet evidence of disc WDs.

As well known, at high Galactic latitude the various Galactic components are expected to contribute to the total CMD with different colours. While the bluer colours are dominated by spheroid stars and the redder object belongs to the disc population, the intermediate colours are dominated by thick disc stars. As a consequence, the whole sample of stars in the CMD appears rather uniformly spreaded, without the “double peaked” feature usually predicted by the two-components models, due to the separation in colour between the disc and the spheroid population.

We are now in the position to explore the role played by the Galactic WDs on the star counts. Figure 10 shows the predicted contribution of each Galaxy component to the V-magnitude distribution of the sample at the intermediate latitude $b = 50^\circ$. An important finding of our prediction is that observations down to $V = 28$ include almost the whole disc population and disc WDs. However, only a few percent of thick disc and spheroid WDs are observable to this luminosities; in fact Fig. 10 shows that the thick disc WD distribution appears centered at $V \sim 30$, while the spheroid

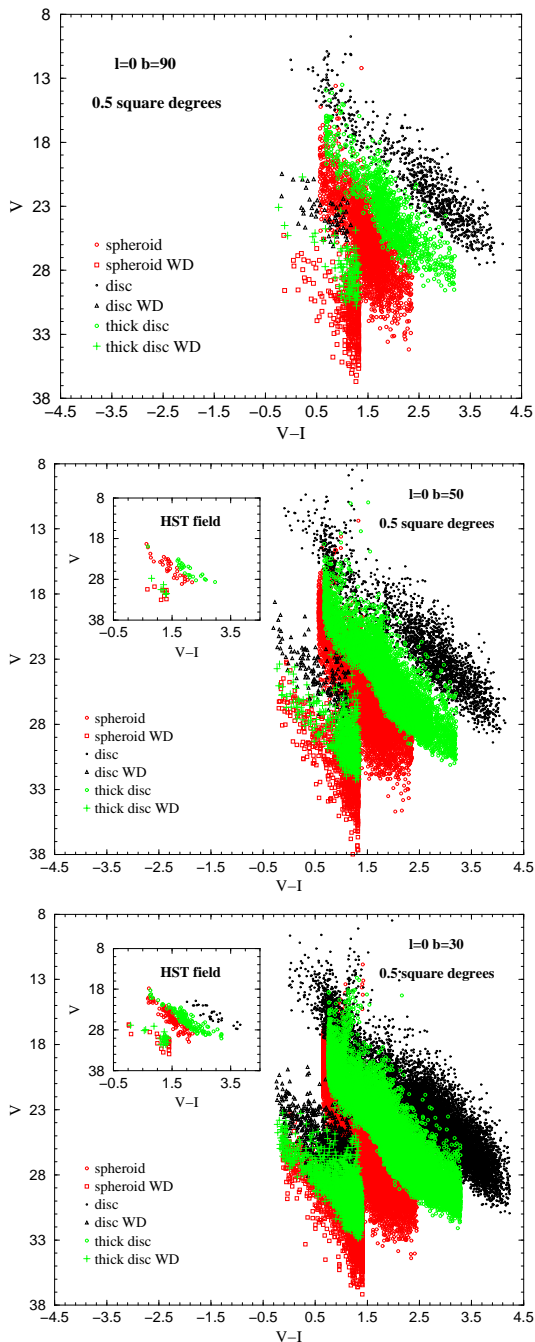


Figure 8. Theoretical $(V-I, V)$ CMDs for field stars in the three Galaxy directions: NGP; $l = 0^\circ$, $b = 50^\circ$; $l = 0^\circ$, $b = 30^\circ$; the area is 0.5 square degrees. Different symbols refer to stars of the various Galaxy populations as labeled; white dwarfs are separately shown. Note that in our model we do not introduce artificial colour dispersion simulating observational spread in colours.

distribution is centered at $V \sim 31$, with a tail reaching faint luminosities down to $V \sim 36$.

Figure 11 (upper panel) shows the predicted $(V-I)$ -colour distributions of the Galaxy components in the apparent magnitude range $19 < V < 27$. The bottom panel is just an enlargement of the figure displaying the behaviour of the WD populations. The whole WD population takes place at colours bluer than $V-I \sim 1.5$. Inspection of this

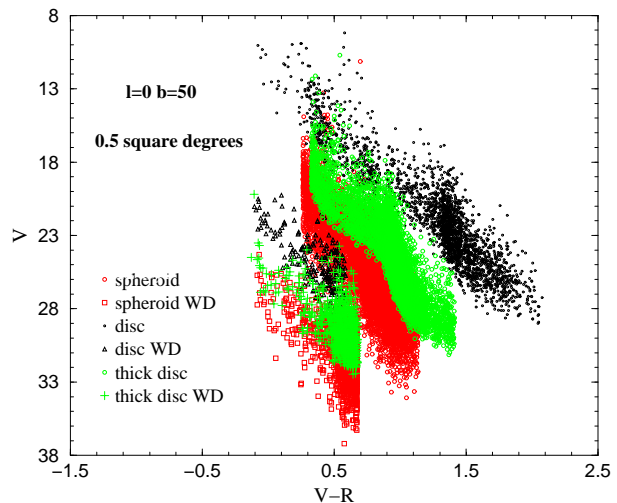


Figure 9. Theoretical $(V-R, V)$ CMD for stars in a field centered at the Galactic coordinates $l = 0^\circ$, $b = 50^\circ$ (area of 0.5 square degrees). Different symbols refer to stars of the various Galactic components, as labeled; white dwarfs are separately shown.

figure reveals that there are regions of the CMDs in which the contribution of white dwarfs to the star counts seems to be distinguishable from other Galaxy stars. In particular, at sufficiently blue colours, i.e. $V-I \lesssim 0.5$, and not too high luminosity ($V \gtrsim 16$), the sample should be constituted exclusively by WDs.

5 CONCLUSIONS

We presented a Galactic model able to reproduce star counts and synthetic colour-magnitude diagrams of field stars from the main sequence to the white dwarf evolutionary phase for various photometric bands and Galactic coordinates. The main goal is the introduction of disc/thick disc and spheroid WD population in an evolutionary consistent way. To this aim we relied on suitable WD mass-progenitor mass relations, theoretical WD models and colour transformations. As a result, we find that the predicted local WD luminosity function appears in *good* agreement with recent observational estimates. Predictions for CMDs and star counts for the various components in different fields of the Galaxy are shown. Luckily enough, we find that the WD population is barely sensitive to a change of theoretical WD models or to a variation of the adopted relation between the WD mass and the progenitor mass while, as expected, the results are significantly affected by the variation of the initial mass function IMF for masses which could evolve into WDs in a time shorter than the estimated age of the Universe.

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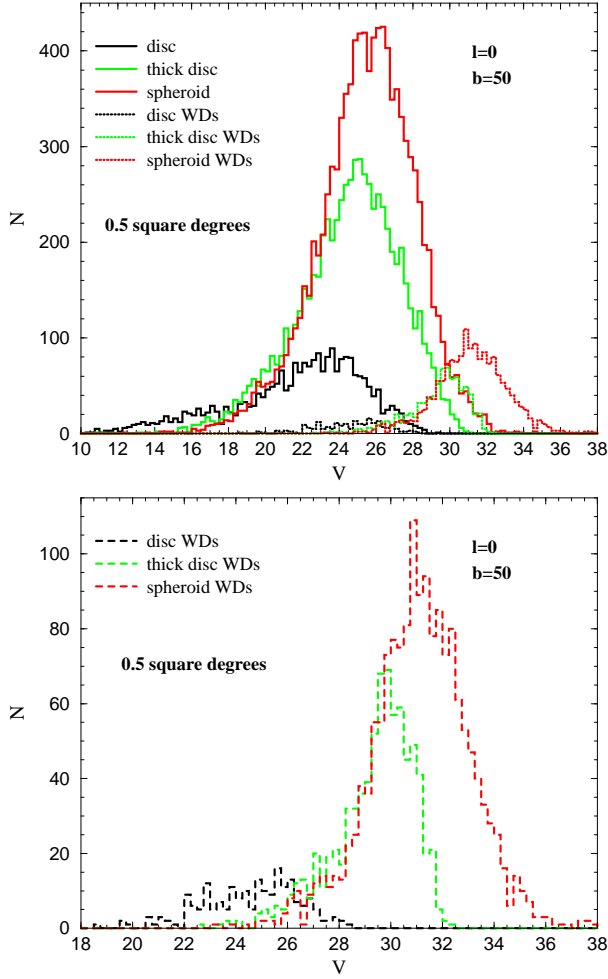


Figure 10. Theoretical apparent- V magnitude distribution of stars in the field of Fig. 9 ($l = 0^\circ$, $b = 50^\circ$). Symbols for the various Galactic populations are labeled. The bottom panel is an enlargement of the upper one. The adopted spheroid age is 12 Gyr.

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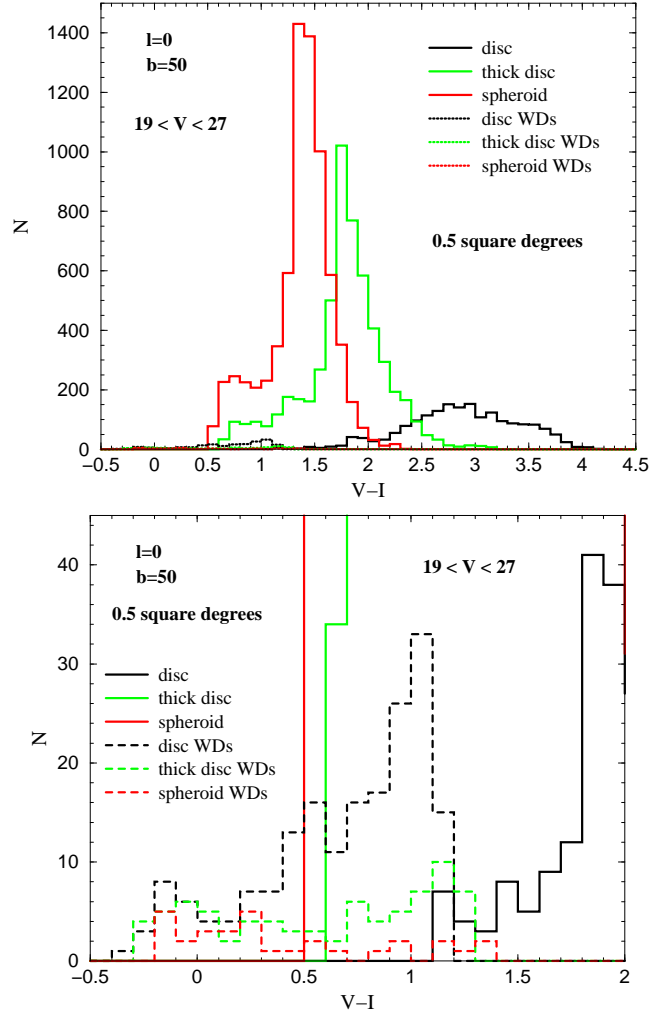


Figure 11. Theoretical $(V - I)$ -colour distribution of stars in the field of Fig. 9 ($l = 0^\circ$, $b = 50^\circ$). Symbols for the various Galactic populations are labeled. The bottom panel is an enlargement of the upper one. Note that in our model we do not introduce artificial colour dispersion simulating observational spread in colours.

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